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(May–June)

FYUP : 4th Semester Examination

MATHEMATICS

(Differential Equations)

(MTH–251)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **four** questions, selecting **one** from each Unit

UNIT—I

1. (a) Obtain the differential equation of all circles passing through the origin and their centres lying on the X-axis. 3

(b) Solve : 5

$$(1+x^2)\frac{dy}{dx} - xy = 1$$

(c) Solve $(6x-8y-5)\,dy = (3x-4y-2)\,dx$. 5

(d) Solve any *two* of the following : 3×2=6

(i) $\sec^2 y \tan x \, dx + \sec^2 x \tan y \, dy = 0$

(ii) $\frac{dy}{dx} + \frac{y}{x} = y^2$

(iii) $p^2 - 7p + 12 = 0$, where $p = \frac{dy}{dx}$

2. (a) Show that $Ax^2 + By^2 = 1$ is the solution of $x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$. 3

(b) Obtain the general solution and singular solution of the equation $y = px + \frac{a}{p}$, where $p = \frac{dy}{dx}$ and a is a constant. 5

(c) Check whether the equation

$$(x^2 + y^2)\,dx - 2xy\,dy = 0$$

is exact. If it is not exact, find the integrating factor and hence solve the equation. 2+1+2=5

(d) Solve any *two* of the following : 3×2=6

(i) $(x^3 - y^3)\,dx + xy^2\,dy = 0$

(ii) $(e^x + 1)y\,dy + (y+1)e^x\,dx = 0$

(iii) $p^2 + 2px - 3x^2 = 0$, where $p = \frac{dy}{dx}$

UNIT—II

3. (a) Solve $(D^3 + 1)y = 0$. 3

(b) Solve any *three* of the following : 5×3=15

(i) $(D^2 + 4)y = \cos 2x$

(ii) $(D^2 - 2D + 2)y = e^x \sin x$

(iii) $(D^2 + 2D + 1)y = x^2 + 2x$

(iv) $(D^3 - 5D^2 + 7D - 3)y = e^{3x}$

4. (a) Solve $(D^3 - 3D + 2)y = 0$. 3

(b) Solve any *three* of the following : 5×3=15

(i) $(D^3 - D^2 - 6D)y = x^2$

(ii) $(D^2 - 2D + 1)y = x^2 e^{3x}$

(iii) $(D^2 + 4)y = x \cos x$

(iv) $(D^2 - 2D + 5)y = 10 \sin x$

UNIT—III

5. (a) Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x \log x$. 4

(b) Reduce the equation

$$\frac{d^2y}{dx^2} - 2\tan x \cdot \frac{dy}{dx} + 5y = e^x \sec x$$

to the normal form and hence solve it. 5

(c) Solve the equation

$$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2y = x^4$$

by changing the independent variable. 5

(d) Solve : 5

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$$

6. (a) Show that the equation

$$x^3\frac{d^3y}{dx^3} + 9x^2\frac{d^2y}{dx^2} + 18x\frac{dy}{dx} + 6y = \cos x$$

is exact and hence solve it. 4

(b) Apply the method of variation of parameters to solve the equation

$$\frac{d^2y}{dx^2} + K^2y = \sec Kx$$

5

(c) Show that the equation

$$3x^2dx + 3y^2dy + (x^3 + y^3 + e^{2z})\,dz = 0$$

is integrable and hence solve it. 5

(d) Solve : 5

$$(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$$

UNIT—IV

7. (a) Form a partial differential equation of the family of ellipsoids

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

by eliminating the arbitrary constants a, b, c. 4

(b) Find the equation of the integral surface of the following partial differential equation passing through the curve $z = 0$, $y = 2x$:

$$(y - z)p + (z - x)q = x - y$$ 5

(c) Find the complete integral by Charpit's method : 5

$$(p^2 + q^2)y = qz$$

(d) Solve : 5

$$p^2 - q^2 = (x^2 - y^2)z$$

8. (a) Form a partial differential equation by eliminating the function f from

$$f(x^2 + y^2 + z^2,\ z^2 - 2xy) = 0$$ 4

(b) Find the complete integral and singular integral of the equation

$$z = px + qy + p^2 + pq + q^2$$ 5

(c) Solve $(mz - ny)p + (nx - lz)q = ly - mx.$ 5

(d) Find a surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1,\ z = 1.$ 5

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